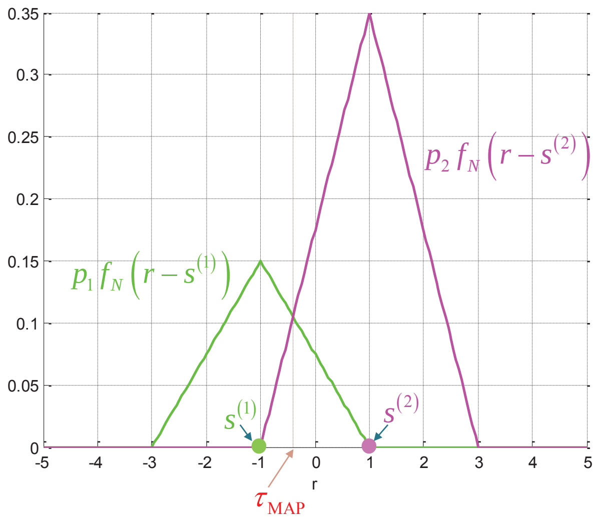
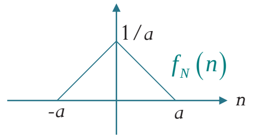
MAP Detector

1. Plot $p_1 f_N(r - s^{(1)}), p_2 f_N(r - s^{(2)}), \ldots, p_M f_N(r - s^{(M)})$.
   - Note that they are functions of r.
2. Select the maximum plot for each (observed) r value.
   - If there are multiple max values, select any.
   - The corresponding $s^{(j)}$ is the value of $\hat{s}_{\text{MAP}}$ at r
3. The area under the max (selected) plot is $P(\mathcal{C})$
   - $P(\mathcal{E}) = 1 - P(\mathcal{C})$

MAP Detector

Ex. Binary PAM under "Triangular" Noise





$$\mathcal{D}_1 = (-\infty, \tau_{\text{MAP}})$$

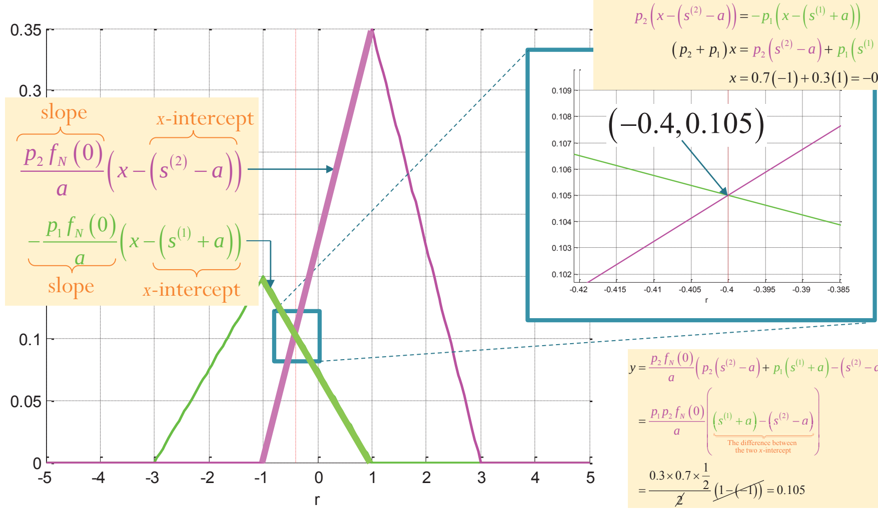
$$\hat{s}_{\text{MAP}}(r) = \begin{cases} s^{(1)}, & r < \tau_{\text{MAP}}, \\ s^{(2)}, & r \geq \tau_{\text{MAP}}. \end{cases}$$

$$\mathcal{D}_2 = [\tau_{\text{MAP}}, +\infty)$$

$$\hat{w}_{\text{MAP}}(r) = \begin{cases} 1, & r < \tau_{\text{MAP}}, \\ 2, & r \geq \tau_{\text{MAP}}. \end{cases}$$

MAP Detector

Ex. Binary PAM under "Triangular" Noise



$$\frac{p_2 f_N(0)}{a}\left(x - \left(s^{(2)} - a\right)\right) = -\frac{p_1 f_N(0)}{a}\left(x - \left(s^{(1)} + a\right)\right)$$

$$p_2\left(x - \left(s^{(2)} - a\right)\right) = -p_1\left(x - \left(s^{(1)} + a\right)\right)$$

$$(p_2 + p_1)x = p_2\left(s^{(2)} - a\right) + p_1\left(s^{(1)} + a\right)$$

$$x = 0.7(-1) + 0.3(1) = -0.4$$

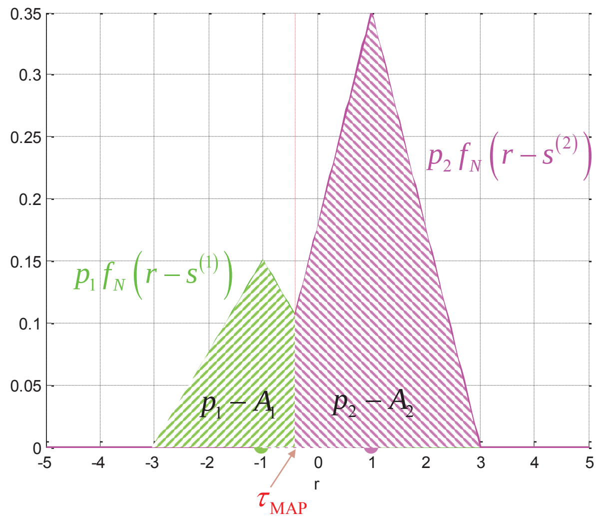
$$y = \frac{p_2 f_N(0)}{a}\left(p_2\left(s^{(2)} - a\right) + p_1\left(s^{(1)} + a\right) - \left(s^{(2)} - a\right)\right)$$

$$= \frac{p_1 p_2 f_N(0)}{a}\left(\underbrace{\left(s^{(1)} + a\right) - \left(s^{(2)} - a\right)}_{\text{The difference between the two } x\text{-intercept}}\right)$$

$$= \frac{0.3 \times 0.7 \times \frac{1}{2}}{2}(1 - (-1)) = 0.105$$

Error Probability

Ex. Binary PAM under "Triangular" Noise

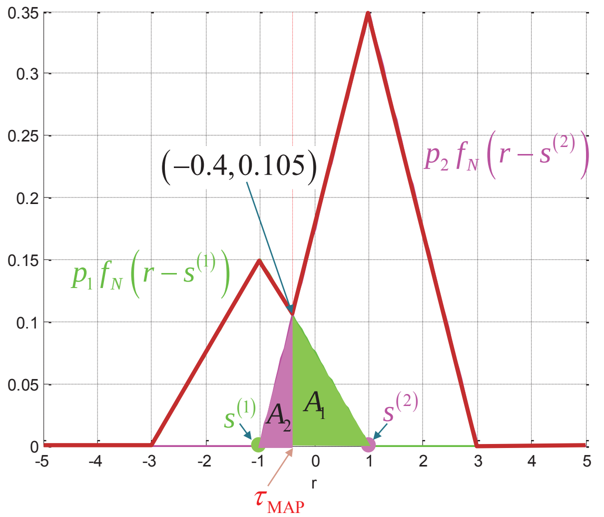


$$P(\mathcal{C}) = (p_1 - A_1) + (p_2 - A_2)$$
$$= 1 - (A_1 + A_2)$$

$$P(\mathcal{E}) = 1 - P(\mathcal{C})$$
$$= A_1 + A_2$$

Error Probability

Ex. Binary PAM under "Triangular" Noise



$$P(\mathcal{C}) = (p_1 - A_1) + (p_2 - A_2)$$
$$= 1 - (A_1 + A_2)$$
$$P(\mathcal{E}) = 1 - P(\mathcal{C})$$
$$= A_1 + A_2$$
$$\approx \frac{1}{2} \times 2 \times 0.105$$
$$= 0.105$$

Error Probability for Arbitrary Detector

$$P(\mathcal{C}) = \sum_{i=1}^{M} \underbrace{\int_{\mathcal{D}_i} p_i f_N\left(r - s^{(i)}\right) dr}_{}$$

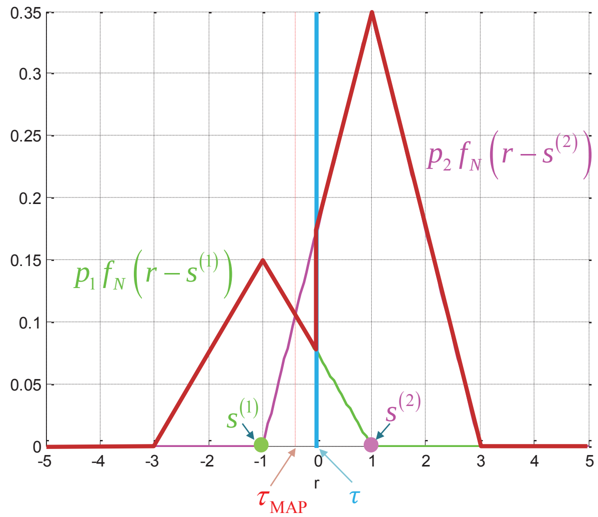
The area under the ith graph inside its detection region.

$$P(\mathcal{E}) = 1 - P(\mathcal{C}) = \sum_{i=1}^{M} \int_{D_i^c} p_i f_N\left(r - s^{(i)}\right) dr$$

The area under the ith graph outside of its detection region.

Error Probability for Arbitrary Detector

Ex. Binary PAM under "Triangular" Noise



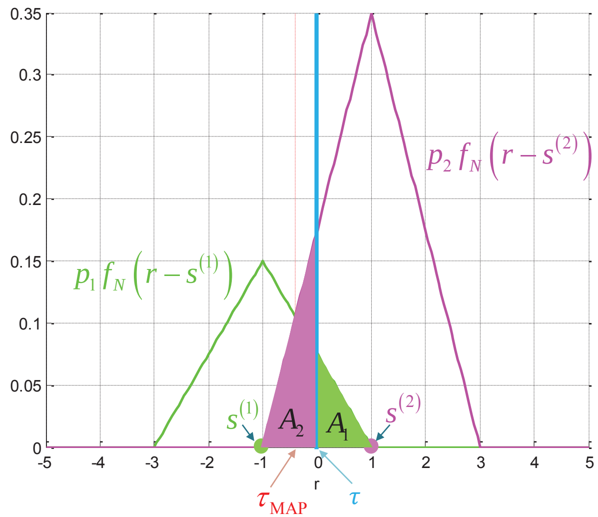
$$\hat{s}(r) = \begin{cases} s^{(1)}, & r < \tau, \\ s^{(2)}, & r \geq \tau. \end{cases}$$

$\mathcal{D}_1 = (-\infty, \tau)$

$\mathcal{D}_2 = [\tau, +\infty)$

Error Probability for Arbitrary Detector

Ex. Binary PAM under "Triangular" Noise



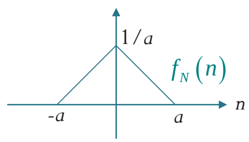
$$P(\mathcal{C}) = (p_1 - A_1) + (p_2 - A_2)$$
$$= 1 - (A_1 + A_2)$$
$$P(\mathcal{E}) = 1 - P(\mathcal{C})$$
$$= A_1 + A_2$$

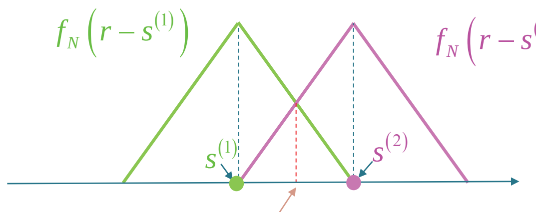
ML Detector

1. Plot $f_N(r-s^{(1)}), f_N(r-s^{(2)}), \ldots, f_N(r-s^{(M)})$.
   - Note that they are functions of r.
   - Note that there is no weighing by the prior probabilities.
2. Select the maximum plot for each (observed) r value.
   - If there are multiple max values, select any.
   - The corresponding $s^{(j)}$ is the value of $\hat{s}_{\text{ML}}$ at r
3. Calculate $P(\mathcal{E})$ from the corresponding detection regions.
   - Note that $P(\mathcal{E})$ is calculated by areas from the plots of $p_1 f_N(r-s^{(1)}), p_2 f_N(r-s^{(2)}), \ldots, p_M f_N(r-s^{(M)})$.

ML Detector

Ex. Binary PAM under "Triangular" Noise





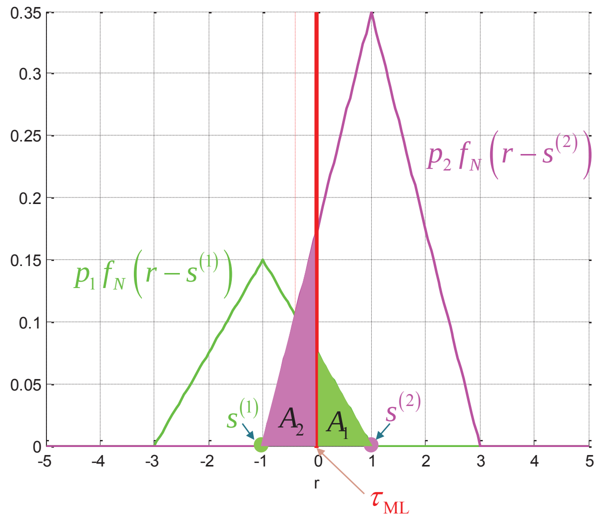
$$\tau_{\text{ML}} = \frac{s^{(1)}+s^{(2)}}{2}$$

$$\hat{s}_{\text{ML}}(r) = \begin{cases} s^{(1)}, & r < \tau_{\text{ML}}, \quad \mathcal{D}_1 = (-\infty, \tau_{\text{ML}}) \\ s^{(2)}, & r \geq \tau_{\text{ML}}. \quad \mathcal{D}_2 = [\tau_{\text{ML}}, +\infty) \end{cases}$$

$$\hat{w}_{\text{ML}}(r) = \begin{cases} 1, & r < \tau_{\text{ML}}, \\ 2, & r \geq \tau_{\text{ML}}. \end{cases}$$

Error Probability for ML Detector

Ex. Binary PAM under "Triangular" Noise

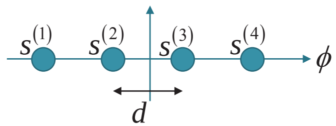


$$P(\mathcal{C}) = (p_1 - A_1) + (p_2 - A_2) = 1 - (A_1 + A_2)$$

$$P(\mathcal{E}) = 1 - P(\mathcal{C}) = A_1 + A_2$$

Note, again, that $P(\mathcal{E})$ is calculated by areas from the plots of $p_1 f_N(r-s^{(1)})$, $p_2 f_N(r-s^{(2)}), \ldots, p_M f_N(r-s^{(M)})$.

Standard Quaternary PAM



$$q = Q\left(\frac{d}{2\sigma}\right) = Q\left(\sqrt{\frac{d^2}{2N_0}}\right) = Q\left(\sqrt{\frac{2E_b}{5\sigma^2}}\right) = Q\left(\sqrt{\frac{4E_b}{5N_0}}\right)$$

$$P(\mathcal{E}|i) = \begin{cases} 2q, & i = 2,3 \\ q, & i = 1,4 \end{cases}$$

$$\underbrace{(\log_2 M)}_{2} E_b = E_s = \frac{1}{4}\left(-\frac{3d}{2}\right)^2 + \frac{1}{4}\left(-\frac{d}{2}\right)^2 + \frac{1}{4}\left(\frac{d}{2}\right)^2 + \frac{1}{4}\left(\frac{3d}{2}\right)^2 = \frac{5}{4}d^2$$

Note: the constellation could be shifted horizontally;
however, the one that is "centered" at origin use minimum E_s.

$$P(\mathcal{E}) = \frac{1}{M}\sum_{i=1}^{4} P(\mathcal{E}|i) = \frac{3}{2}q = \frac{3}{2}Q\left(\sqrt{\frac{4E_b}{5N_0}}\right)$$